Overview of Maurice A. Biot's 1956 paper on:
Theory of propagation of elastic waves in a fluid-saturated porous solid I. Low-frequency Range

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INCE USA

Background

Maurice A. Biot

- Education[2]
    - Leuven (Belgium):
        - Bachelors: Philosophy (1927)
        - Bachelors: Mining Engineering (1929)
        - Bachelors: Electrical Engineering (1930)
        - Doctor of Science (1931)
    - CIT (USA):
        - Ph.D.: Aeronautical Science (1932)
- Scientific Contributions[2]
    - Aircraft flutter
    - RSM in earthquake engineering
    - Thermodynamic dissipation
    - **Poromechanics, Poroelasticity**
- Legacy
    - Maurice A. Biot Medal (ASCE)
    - Biot Conference on Poromechanics

Maurice A. Biot in 1964

Objective of 1956 Paper

- Previous work assumed a rigid solid, or ignored certain wave types[1],[4]

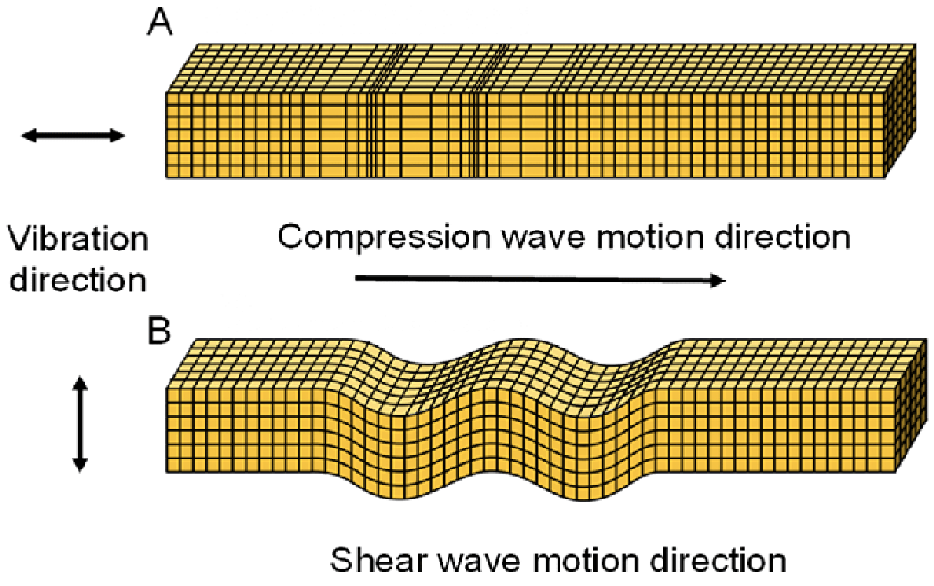
"Our purpose is to establish a theory of propagation of elastic waves in a system composed of a porous elastic solid saturated by a viscous fluid."[1]

- Interest in:
  - Seismology
  - Osteology
  - **Passive noise control**

System Details and Assumptions

- Two-phase porous material
  - Comparable solid and fluid density
  - Elastic solid
  - Compressible, viscous fluid
- Dilatational & rotational elastic waves present
- Poiseuille flow assumption
  - Low frequency, limit depends on pore size

$$f_t = \frac{\pi \nu}{4d^2}$$



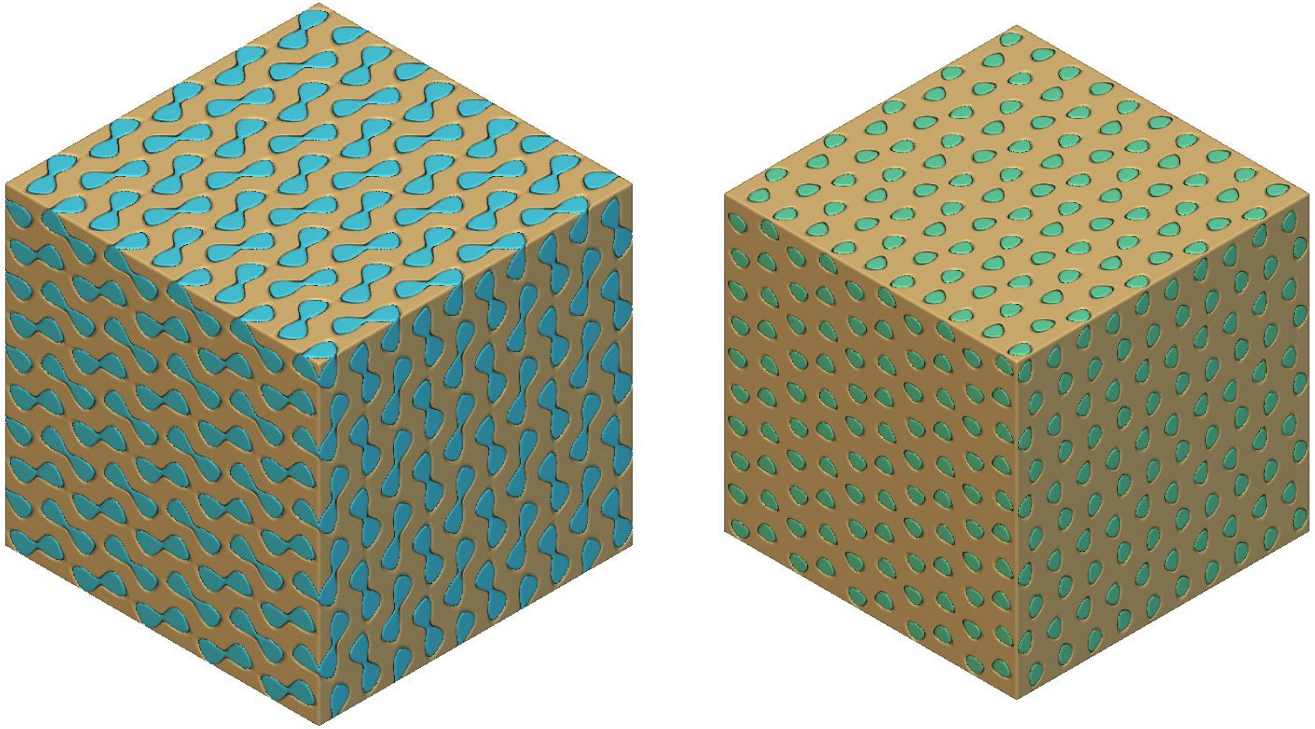
Elastic waves[2]

Solid and Fluid Strain Relationship

- The strain of the solid (e) and fluid (ϵ) are of opposite sign

$$\epsilon = -\frac{Q}{R}e$$

- Familiar example: squeezing a sponge



Non-Dissipative Propagation

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Governing Propagation Equations

P,Q, R: elastic coefficients

ρ: related to mass densities

Solid (e, $\vec{\omega}$) and fluid (ϵ, $\vec{\Omega}$) reactions are present in every equation:

Waves contain coupled motion

Dilatational Waves:

$$\nabla^2(Pe + Q\epsilon) = \frac{\partial^2}{\partial t^2}(\rho_{11}e + \rho_{12}\epsilon)$$

$$\nabla^2(Qe + R\epsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}e + \rho_{22}\epsilon)$$

Rotational Waves:

$$\frac{\partial^2}{\partial t^2}\left(\rho_{11}\vec{\omega} + \rho_{12}\vec{\Omega}\right) = N\nabla\vec{\omega}$$

$$\frac{\partial^2}{\partial t^2}\left(\rho_{12}\vec{\omega} + \rho_{22}\vec{\Omega}\right) = 0$$

Solving the Equations; Finding Waves

One rotational wave

- Propagation velocity V_s

$$V_s = \sqrt{\frac{N}{\rho_{11}\left(1 - \frac{\rho_{12}^2}{\rho_{11}\rho_{22}}\right)}}$$

Two dilatational waves

- A result of two roots (z_1, z_2)
- High-velocity: “Wave of the first kind”
- Low-velocity: “Wave of the second kind”

$$V_1^2 = \frac{V_c^2}{z_1}$$

$$V_2^2 = \frac{V_c^2}{z_2}$$

Dynamic Compatibility Case

No relative motion between solid and fluid, where:

$$\frac{\sigma_{11}+\sigma_{12}}{\gamma_{11}+\gamma_{12}}=\frac{\sigma_{22}+\sigma_{12}}{\gamma_{22}+\gamma_{12}}=1$$

- σ_{ij} terms define elastic properties
- γ_{ij} terms define dynamic properties

No relative motion results in No fluid friction dissipation

Later in the paper, Biot nondimensionalizes frequencies and velocities with respect to this condition.

Dissipative Propagation

Governing Propagation Equations

b is a dissipative coefficient:

$$b = \frac{\mu\beta^2}{k}$$

Dissipative terms are bold.

<u>Dilatational Waves:</u>

$$\nabla^2(Pe + Q\epsilon) = \frac{\partial^2}{\partial t^2}(\rho_{11}e + \rho_{12}\epsilon) + \mathbf{b}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}(\boldsymbol{e} - \boldsymbol{\epsilon})$$

$$\nabla^2(Qe + R\epsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}e + \rho_{22}\epsilon) - \mathbf{b}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}(\boldsymbol{e} - \boldsymbol{\epsilon})$$

<u>Rotational Waves:</u>

$$\frac{\partial^2}{\partial t^2}\left(\rho_{11}\vec{\omega} + \rho_{12}\vec{\Omega}\right) + \boldsymbol{b}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}(\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\Omega}}) = N\nabla^2\vec{\omega}$$

$$\frac{\partial^2}{\partial t^2}\left(\rho_{12}\vec{\omega} + \rho_{22}\vec{\Omega}\right) - \boldsymbol{b}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial t}}(\vec{\boldsymbol{\omega}} - \vec{\boldsymbol{\Omega}}) = 0$$

Investigating Example Cases

Biot uses example cases of material parameters:

- Cases 2 and 5 have dynamic compatibility

Characteristic frequency and velocities:

- Frequency $f_c = \frac{b}{2\pi\rho(\gamma_{12}+\gamma_{22})}$
- Phase velocity $v_r = {}^{\alpha}/_{|l_2|}$
- Reference velocities $V_c = \sqrt{{}^{P+R+2Q}/_{\rho}}$ and $V_r = \sqrt{{}^{N}/_{\rho}}$

TABLE I.

Case	σ_{11}	σ_{22}	σ_{12}	γ_{11}	γ_{22}	γ_{12}	z_1	z_2
1	0.610	0.305	0.043	0.500	0.500	0	0.812	1.674
2	0.610	0.305	0.043	0.666	0.333	0	0.984	1.203
3	0.610	0.305	0.043	0.800	0.200	0	0.650	1.339
4	0.610	0.305	0.043	0.650	0.650	−0.150	0.909	2.394
5	0.500	0.500	0	0.500	0.500	0	1.000	1.000
6	0.740	0.185	0.037	0.500	0.500	0	0.672	2.736

Rotational Waves

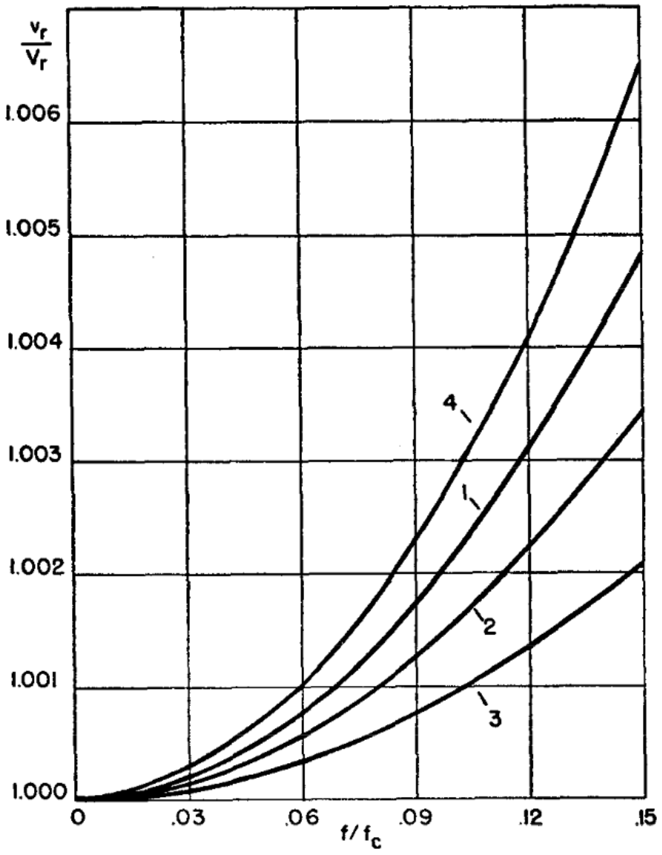

FIG. 1. Phase velocity v_r of rotational waves.

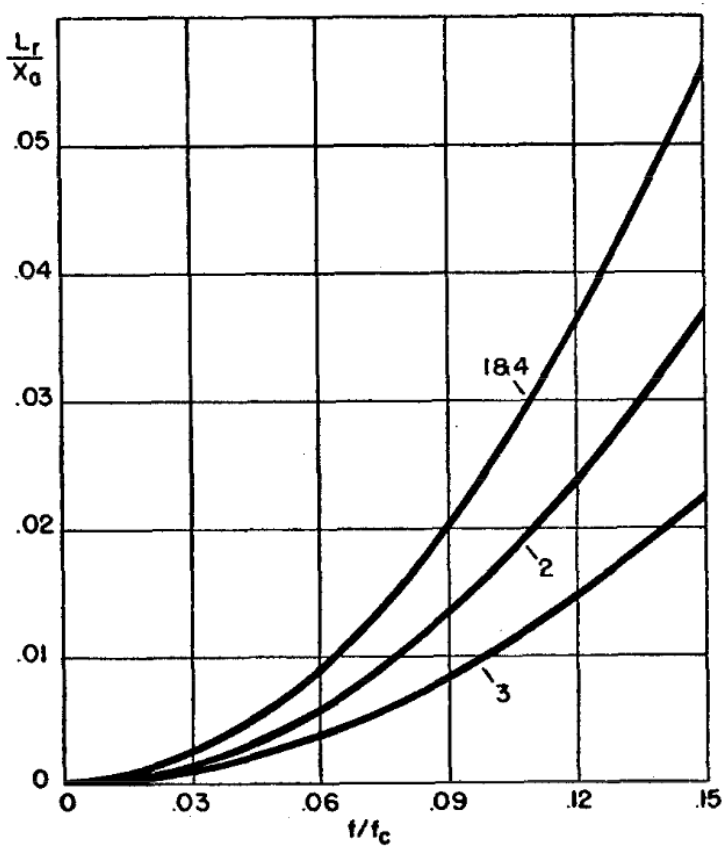

FIG. 2. Attenuation coefficient of rotational waves.

- υ has small variance from υ_r
- $\upsilon \propto f^2$
- Low attenuation
- Attenuation $\propto f^2$

First Kind Dilatational Waves

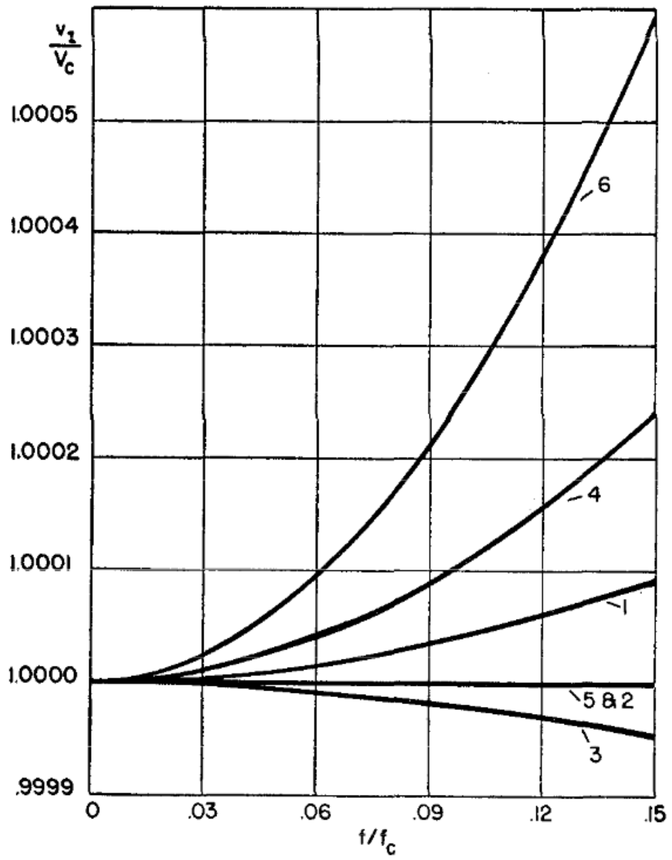

FIG. 3. Phase velocity v_I of dilatational waves of the first kind.

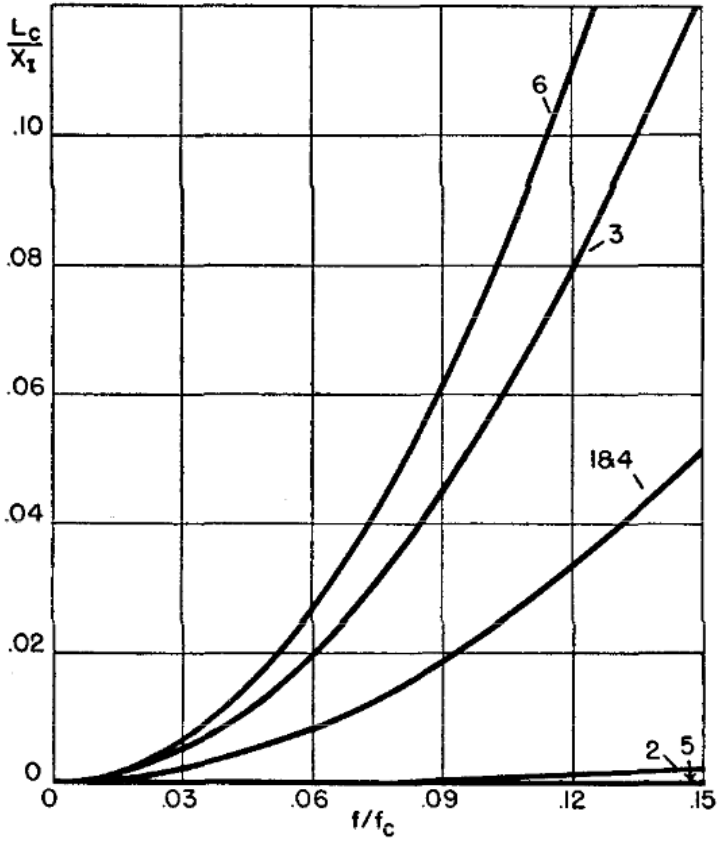

FIG. 4. Attenuation coefficient of dilatational waves of the first kind.

- υ small variance from V_c
  - $\upsilon = V_c$ for dynamic compatibility cases
- $\upsilon \propto f^2$
- Low attenuation
  - ≈ 0 for dynamic compatibility cases
- Attenuation $\propto f^2$

Second Kind Dilatational Waves

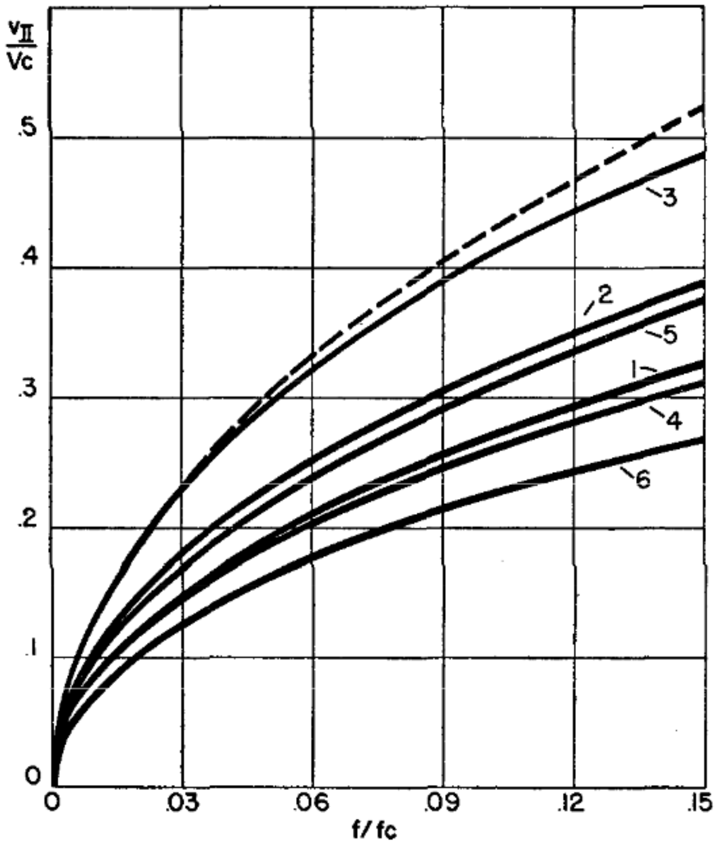

FIG. 5. Phase velocity v_{II} of dilatational waves of the second kind.

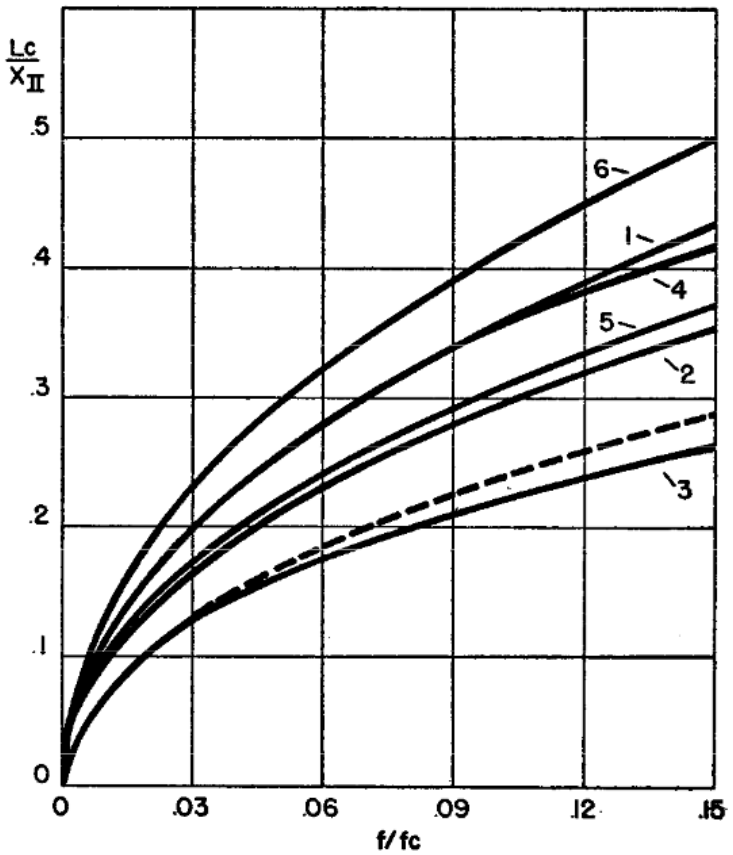

FIG. 6. Attenuation coefficient of dilatational waves of the second kind.

- υ large variance from V_c
  - $\upsilon \neq V_c$ for dynamic compatibility cases
- $\upsilon \propto \sqrt{f}$
- High attenuation
  - $\neq 0$ for dynamic compatibility cases
- Attenuation $\propto \sqrt{f}$

Key Takeaways

- Solid-fluid coupling must be considered in dynamic poroelasticity analysis.
- The dynamic compatibility case has no fluid friction losses and is useful for nondimensionalization.
- The Poiseuille flow assumption can only be used for low frequencies, limit depends on viscosity and pore size.
- One shear/rotational and **two compressive/dilatational** waves are found with different phase velocity and attenuation relationships.

Further Work & Impact

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Investigating Higher Frequencies

Biot's subsequent paper was published simultaneously[5]

- Complex viscosity correction function $F(\kappa)$ from Poiseuille flow
- Including pore complexity effect with 'structure factor' δ

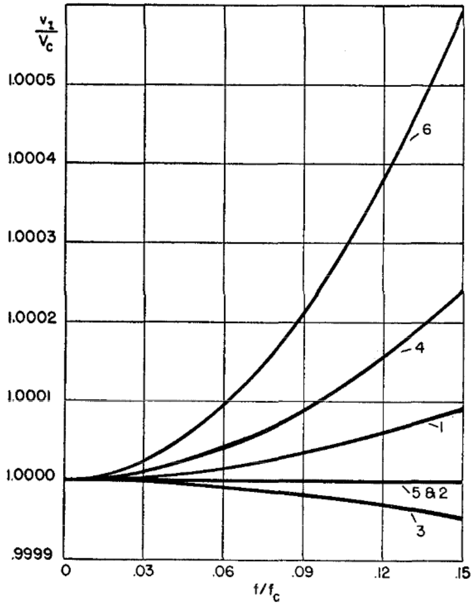
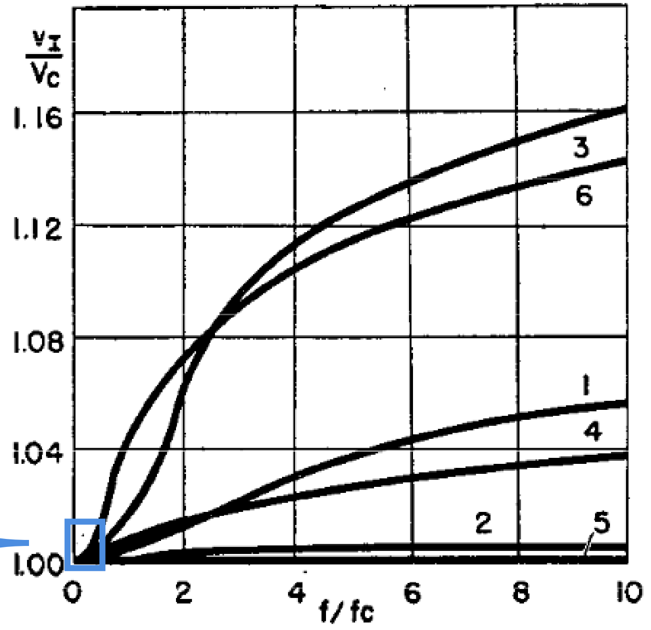
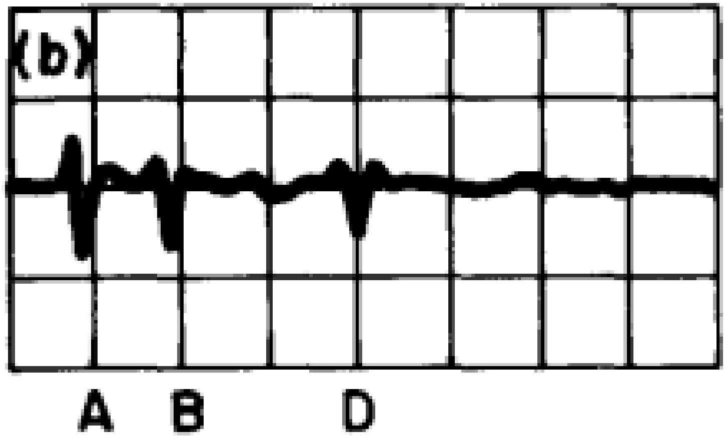


FIG. 3. Phase velocity v_I of dilatational waves of the first kind.



Finding the Slow Dilatational Wave

- Previously observed but not identified
  - “It is shown that two waves travel through the material—one primarily airborne and the other primarily structure-borne”[4]
- Positively identified by Plona (1980) in water-submerged sintered glass beads[6]
  - The slow wave (D) was 4-5 times slower than the fast wave (A)



Semi-phenomenological Models

Predicting macro-level acoustic response from micro-level material parameters

- Johnson et al. proposed a model considering visco-inertial effects[7] using Biot's wave equations
- Champoux and Allard extended the model considering thermal effects[8]
- The JCA and improved[9] versions are popular motionless skeleton models[10]

Symbol	Parameter
ϕ	Porosity
σ	Flow Resistivity
α_∞	Tortuosity
Λ	Viscous Characteristic Length
Λ'	Thermal Characteristic Length

References

[1]: M. A. Biot; Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid. I. Low-Frequency Range. J Acoust Soc Am 1 March 1956; 28 (2): 168–178

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[6]: Thomas J. Plona; Observation of a second bulk compressional wave in a porous medium at ultrasonic frequencies. Appl. Phys. Lett. 15 February 1980; 36 (4): 259–261

[7]: Johnson, D., Koplik, J., & Dashen, R. (1987). Theory of dynamic permeability and tortuosity in fluid-saturated porous media. Journal of Fluid Mechanics, 176, 379-402

[8]: Yvan Champoux, Jean-F. Allard; Dynamic tortuosity and bulk modulus in air-saturated porous media. Journal of Applied Physics 15 August 1991; 70 (4): 1975–1979

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[10]: Ljaouen, Luc. "Johnson-Champoux-Allard (JCA) Model." APMR, Matelys, https://apmr.matelys.com/PropagationModels/MotionlessSkeleton/JohnsonChampouxAllardModel.html

Thank You!

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